Theory of the BAT permeability test

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The permeability test system arrangement for in situ measurement of the hydraulic conductivity (or permeability) is shown in figure 1. The measuring system comprises a test adapter that is equipped with a double-sided hypodermic needle and a gas/water container. The pressure in the container is measured with the aid of an electronic pressure transducer.
The test can be carried out either as an “inflow test” or as an “outflow test”. In the former case the gas/water container is completely gas-filled at the start of the test. An inflow test can be conducted simultaneously with the extraction of a pore water sample. In an outflow test the container is partly filled with water and partly filled with compressed gas.
The test arrangement for an outflow test can also be used for the controlled injection of a tracer liquid into the soil. The spreading of this liquid can be checked by repeated sampling in filter tips installed at different distances from the point of injection.

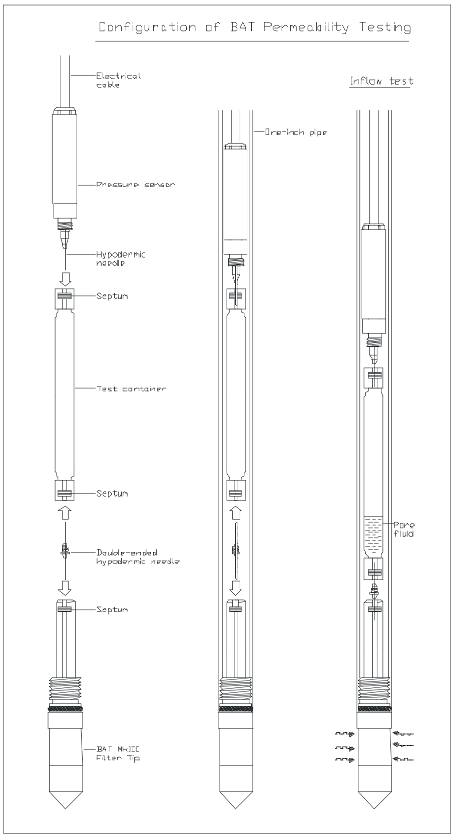

Figure 1: Configuration of BAT permeability Testing

The initial pressure in the gas/water container and the equilibrium pore pressure in the soil are denoted as p_b and p_{gr}, respectively. Before the start of the test, the pore pressure is measured in the conventional manner.
After preparation, the test adapter is lowered down the extension pipe. Temperature equalisation is achieved before connecting the adapter to the filter tip. When the adapter is lowered on the nozzle in the filter tip, it is automatically connected to the tip with the aid of the double-sided hypodermic needle. Upon connection of the test adapter to the filter tip, the pressure in the container starts to change. The change in pressure is recorded with the same electronic pressure transducer.

Parameters in this paper:

g	gravitational acceleration [m/s^2]	
r	specific mass of water [kg/m^3]	
V_d	extra volume [m^3]	
V_c	container volume [m^3]	
V_t	total volume [m^3]	$V_t = V_c + V_d$
V_w	water volume [m^3]	
V_0	gas volume [m^3]	$V_t = V_w + V_0$

$V_{w\ end}$	water volume at the end of the permeability test [m^3]	
$V_{w\ min}$	minimum water volume for an outflow test [m^3]	
V_{in}	inflow water volume or outflow water volume (negative) [m^3]	
A_{con}	area of the container [m^2]	
p	pressure [Pa]	
p_{gr}	pore pressure [Pa]	
p_b	pressure at the beginning of the test [Pa]	
p_{end}	pressure at the end of the test [Pa]	
p_w	pressure at the filter exterior [Pa]	
h	correction height between filter and measurement level [m]	
p_h	correction pressure [Pa]	$p_h = h\ g\ r$
t_{kar}	characteristic time of the permeability test [s]	
k	permeability of the soil [m^4/Ns]	
k'	permeability of the soil [m/s]	$k' = k\ g\ r$
k_{eff}	effective permeability of the soil [m^4/Ns]	
R_l	linear flow resistance [m^5/Ns]	
$R_{nl1}\ R_{nl2}\ R_{nl3}$	non-linear flow resistance parameters [m^5/Ns], [m^3/s], [1/Pa]	
d	diameter of filter [m]	
l	length of filter [m]	
F	flow factor [m]	
Q	water flow [m^3/s]	

Soil

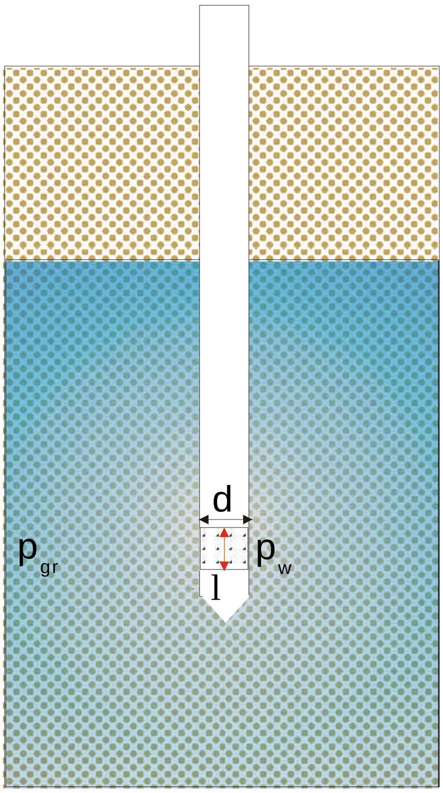

Figure 2

In general, the water pressure in a measurement system differs from the surrounding soil.
This will result in a water flow of:

$$Q = F\ k\ (p_{gr} - p_w) \qquad \text{I}$$

The flow factor according to Horslev[i] for a well point or hole extended in uniform soil is:

$$F = \frac{2\,p\ l}{\ln\left(\frac{l}{d} + \sqrt{1 + \left(\frac{l}{d}\right)^2}\right)} \qquad \text{II}$$

Container model

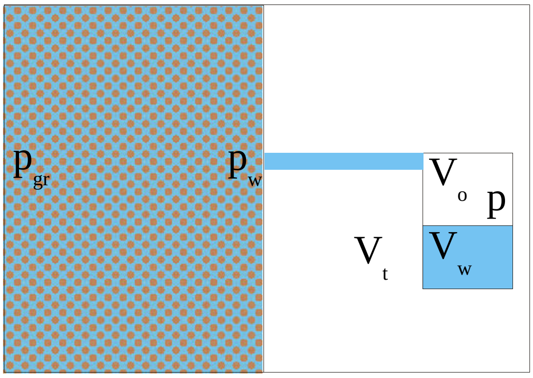

Figure 3

For the gas in the container Boyle's (or Mariotte's) law can be applied:

$$pV = C$$

The flow of water into the container will change the gas volume. This volume change is the integration of the (in or out) flow of water. The connection between the well point and the container is without resistance, which makes $p = p_w$. This results in :

$$p\left(V_0 - \int_{t_0}^{t} F\,k\left(p_{gr} - p\right)dt\right) = p_b V_0 \qquad \text{III}$$

This results in the differential equation:

$$\frac{F\,k}{p_b V_0}\,dt = \frac{-1}{p^3 - p_b\,p^2}\;dp \qquad \text{IV}$$

This equation can be solved[ii] :

$$t = \frac{V_0}{k\,F\,p_{gr}}\,p_b\left(\frac{1}{p_b} - \frac{1}{p} + \frac{1}{p_{gr}}\ln\left(\frac{p_b - p_{gr}}{p_b}\,\frac{p}{p - p_{gr}}\right)\right) \qquad \text{V}$$

This solution was first given by Bengtsson[iii] .The start condition is $p=p_b$ at $t= 0$.

An approximation can be obtained from formula V for small pressure differences:

$$p = p_{gr} - \left(p_{gr} - p_b\right)\exp\left(-t\,\frac{k\,F\,p_{gr}}{V_0}\right) \qquad \text{Va}$$

which defines the characteristic time of the in or out flow test:

$$t_{kar} = \frac{V_0}{k\,F\,p_{gr}} \qquad \text{Vb}$$

End condition

The condition of the gas in the container before and after the permeability test must fulfil Boyle's law (The time in formule Vis infinity):

$$p_b V_0 = p_b (V_t - V_w) = p_{gr} (V_t - V_{w\,end}) \qquad \text{VI}$$

The water volume in the container after the test is

$$V_{w\,end} = V_t \left(1 - \frac{p_b}{p_{gr}}\right) + \frac{p_b}{p_{gr}} V_w \qquad \text{VII}$$

The inflow or outflow water volume after the test is:

$$V_{in} = (V_t - V_w)\left(1 - \frac{p_b}{p_{gr}}\right) \qquad \text{VIII}$$

Inflow

$p_b < p_{gr}$ will result in an inflow test.

Outflow

$p_b > p_{gr}$ will result in an outflow test (with a negative V_{in} in formule VIII). To prevent gas being injected in the soil during an outflow test the minimum volume of water after the test is zero: $V_{w\,end}=0$. The minimum volume of water at the start of the test to prevent the injection of gas must be:

$$V_{w\min} = V_t \left(1 - \frac{p_{gr}}{p_b}\right) \qquad \text{IIX}$$

This results in following requirements for the water or the pressure in the container:

$$V_w \geq V_{w\min} \qquad or \qquad p_b \leq p_{gr}\left(\frac{V_t}{V_t - V_w}\right) \qquad \text{IX}$$

Container model with flow restriction

Linear

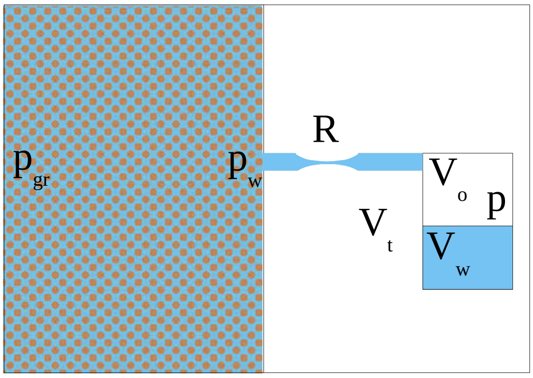

Figure 4

The model can be extended with a linear flow restriction for the resistance caused by the needle:

$$Q = F\,k\left(p_{gr} - p_w\right) \qquad \textit{and} \qquad Q = R_l\,\left(p_w - p\right) \qquad \text{X}$$

Both flows must be equal. p_w can be removed from both equations. This results for the flow of water in:

$$Q = \frac{R_l\,k\,F}{R_l + F\,k}\left(p_{gr} - p\right) \qquad \text{XI}$$

Comparing this result with formula I, which is without flow resistance in the connection to the container, a new effective permeability can be defined:

$$k_{eff} = \frac{R_l\,k}{R_l + F\,k} \qquad \text{XII}$$

This effective permeability can be used to replace the *k* in formula V. This results in the solution for the mathematical model with a linear flow restriction.

Non-linear

Investigation of the flow behaviour of the needle has shown that the flow-pressure characteristic is a non-linear relation. Results of these measurements are shown in figure 5.

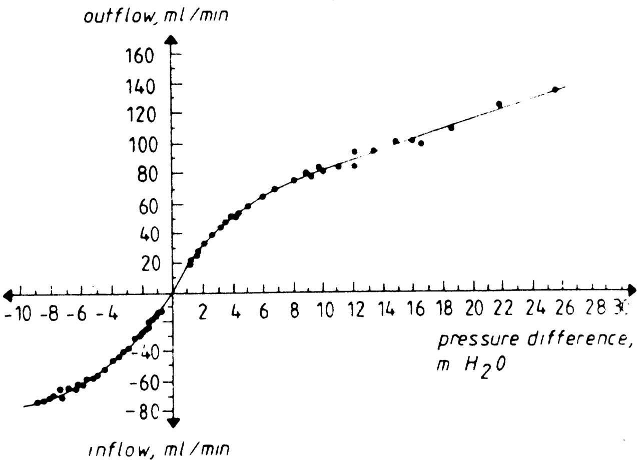

Figure 5: Flow characteristics of the hypodermic needle used in the BAT permeability test system.The figure is copied from Torstensson[iii].

This relation can be approximated by:

$$Q = R_{nl1}\,p + R_{nl2}\,\arctan\left(R_{nl3}\,p\right) \qquad \text{XIII}$$

This relation is not derived from fundamental physical flow properties but is only used because it can describe the measurements from figure 5 with three parameters. This equation is more efficient for calculation purposes than a tabulated function. Formula XIII is shown in figure 6. :

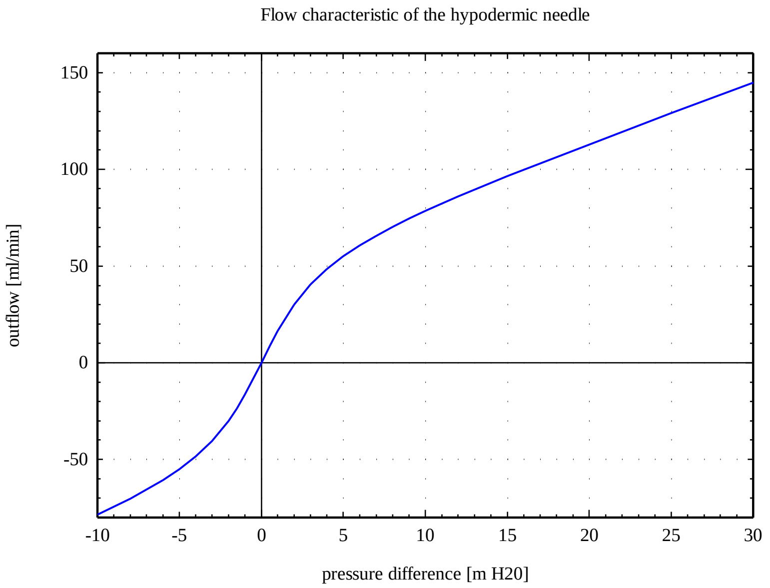

Figure 6:Flow through the hypodermic needle as a function of the pressure difference.

The linear part of the flow characteristic is given by:

$$R_l = R_{nl1} + R_{nl2} R_{nl3} \qquad \text{XIV}$$

A new non-linear differential equation can be obtained for the approximated needle characteristic. The continuity of the flow results in (see formula X, XI):

$$F\,k\left(p_{gr} - p_w\right) = R_{nl1}\left(p_w - p\right) + R_{nl2}\ \arctan\left(R_{nl3}\left(p_w - p\right)\right)$$

This results in a function *g(x):*

$$p_w = g(p) \quad with \quad p_{gr} \geq p_w \geq p \quad or \quad p_{gr} \leq p_w \leq p \qquad \text{XV}$$

With Boyle's law this results in:

$$p\left(V_0 - \int_{t_0}^{t} F\,k\left(p_{gr} - g(p)\right)dt\right) = p_b V_0 \qquad \text{XVI}$$

The new differential equation is:

$$\frac{F\,k}{p_b V_0}\,dt = \frac{-1}{p^2 g(p) - p_b\,p^2}\ dp \qquad \text{XVII}$$

Container model with water height correction

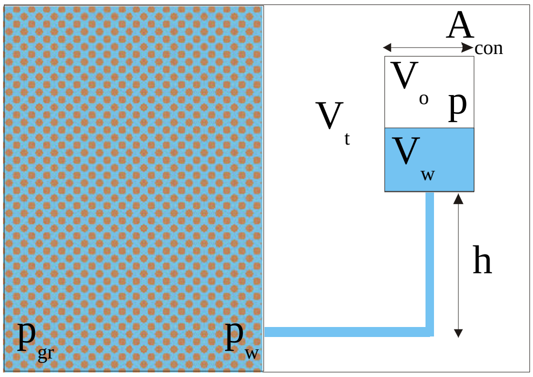

Figure 7

The model for the BAT permeability test system can be extended with a correction for pressure caused by the water column to the container and the water in the container:

$$p_w = p + hr\ g + \frac{V_w}{A_{con}} r\ g \qquad \text{XVIII}$$

This can be rewritten as:

$$p_w = p + \frac{\left(V_t - V_0 \frac{p_b}{p}\right) r\ g}{A_{con}} + p_h \qquad \text{XIIX}$$

This results with formula I and III in:

$$p\left(V_0 - \int_{t_0}^{t} F\ k \left(p_{gr} - \frac{V_t r\ g}{A_{con}} + \frac{p_b}{p}\frac{V_0 r\ g}{A_{con}} - p_h - p\right) dt\right) = p_b V_0 \qquad \text{XIX}$$

This results in the differential equation:

$$\frac{F\ k}{p_b V_0} dt = \frac{1}{\left(p_{gr} - \frac{V_t r\ g}{A_{con}} - p_h\right) p^2 - p^3 + p_b \frac{V_0 r\ g}{A_{con}} p} dp \qquad \text{XX}$$

This equation can be solved[ii]:

$$t = \frac{V_0}{k\ F\ p_{gr}} p_{gr} p_b \left(-K_1 \ln\left(\frac{p}{p_b}\right) + K_2 \ln\left(\frac{p - a_1}{p_b - a_1}\right) + K_3 \ln\left(\frac{p - a_2}{p_b - a_2}\right)\right) \qquad \text{XXI}$$

With

$$p_1 = p_{gr} - \frac{V_t r\ g}{A_{con}} - p_h \quad and \quad p_2 = p_b \frac{V_0 r\ g}{A_{con}} \qquad \text{XXIa}$$

and

$$a_1 = \frac{p_1 + \sqrt{p_1^2 + 4p_2}}{2} \quad and \quad a_2 = \frac{p_1 - \sqrt{p_1^2 + 4p_2}}{2} \qquad \text{XXIb}$$

and

$$K_1 = \frac{1}{p_2} \quad and \quad K_2 = \frac{1}{a_1^2 + p_2} \quad and \quad K_3 = \frac{1}{a_2^2 + p_2} \qquad \text{XXIc}$$

The start condition for equation XXI is $p=p_b$ at $t= 0$.

End condition

The water volume at the end of the test can be obtained from Boyles law (time in formula XXI is infinity.):

$$p_b(V_t - V_w) = p_{end}(V_t - V_{w\,end}) = \left(p_{gr} - p_h - \frac{V_{w\,end}\,gr}{A_{con}}\right)(V_t - V_{w\,end}) \quad \text{XXVI}$$

$V_{w\,end}$ can be obtained from formula XXVI:

$$V_{w\,end} = \frac{b - \sqrt{b^2 - 4ac}}{2a} \qquad \text{XXVIII}$$

with:

$$a = \frac{gr}{A_{con}} \quad -b = p_{gr} - p_h + \frac{V_t gr}{A_{con}} \quad c = p_{gr}V_t - p_hV_t - p_bV_t + p_bV_w \qquad \text{XXVIIIa}$$

The inflow or outflow of water is:

$$V_{in} = (V_w - V_{w\,end}) \qquad \text{XXIIX}$$

Inflow

The condition for an inflow test is:

$$p_{gr} > p_b + p_h + \frac{V_w r\ g}{A_{con}} \qquad \text{XXII}$$

Since $p_b>0$ the folowing condition must be fulfiled:

$$p_{gr} > p_h + \frac{V_w r\ g}{A_{con}} \qquad \text{XXIII}$$

Outflow

The minimum value for $V_{w\,end}$ is zero, to prevent gas from being injected during an outflow test. The minimum volume of water at the beginning of the test is (from formula XXVI):

$$V_{w\min} = V_t\left(1 - \frac{p_{gr} - p_h}{p_b}\right) \qquad \text{XXIV}$$

Which results in following conditions for the water volume or the pressure in the container at the beginning of the permeability test:

$$V_w \geq V_{w\min} \qquad or \qquad p_b \leq \left(p_{gr} - p_h\right)\left(\frac{V_t}{V_t - V_w}\right) \qquad \text{XXV}$$

Container model with water height correction and flow restriction

Formula XXI can be extended with a flow restriction. The k_{eff} will give the result for a linear flow restriction. (See also equation: XII)

The differential equation for the non-linear flow restriction can be obtained by the same reasoning as in formula XV.

Appendix

A software program is available for the calculation of the formulas in this paper. The numerical values in the Permeability Test Software version 0.91 are (*Remark: these values cannot be changed in the software program.*):

g	9,81 [m/s^2]	
r	1000 [kg/m^3]	
V_d	0,0 [1e-6].[m^3]	
V_c	36 or 72 [1e-6] [m^3]	
A_{con}	2,0 [1e-4] [m^2]	
h	0,223 [m]	
R_{nl1}	3.00 [1/60. 1e-6 / gr] [m^5/Ns]	
R_{nl2}	36,9 [1/60. 1e-6] [m^3/s]	
R_{nl3}	0,38 [1/ gr] [1/Pa]	
d	0,031 [m]	(Mark III filter tip)
l	0,036 [m]	

[i] Horslev. M.J. 1951. Time lag and soil permeability in ground water observations. Corps of Engineers. Waterways Experiment Station. Vickburg. Mississippi. Bull. 36,50 pp.
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[ii] Handboek der wiskunde 1963 Scheltema & Holkema N.V. Amsterdam page 156..159

[iii] Torstensson, B-A. 1978, The pore pressure probe. Proc. Geoteknikkedagen, pp. 34.1-34-15, Tapir forlag, Oslo, Norway. Bengtsson. P.E. 1984. Personal communication